

# Metallic and Insulating behaviour in p-SiGe at $\nu = 3/2$

P.T. Coleridge, R.L. Williams, J. Lapointe and P. Zawadzki  
*Institute for Microstructural Sciences  
National Research Council  
Ottawa, Ontario, K1A 0R6, Canada*

Shubnikov-de Haas data is presented for a p-SiGe sample exhibiting strongly insulating behaviour at  $\nu = 3/2$ . In addition to the fixed points defining a high field metal-insulator transition into this phase separate fixed points can also be identified for the  $\nu = 3 \rightarrow 2$  and  $2 \rightarrow 1$  Integer quantum Hall transitions. Another feature of the data, that the Hall resistivity approaches zero in the insulating phase, indicates it is not a re-entrant Hall insulator. The behaviour is explained in terms of the strong exchange interactions. At integer filling factors these cause the  $0 \uparrow$  and  $1 \downarrow$  Landau levels to cross and be well separated but at non-integer values of  $\nu$  screening reduces exchange effects and causes the levels to stick together. It is suggested the insulating behaviour, and high field metal/insulator transition, is a consequence of the strong exchange interactions.

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## I. INTRODUCTION

The appearance, in p-SiGe, of insulating behaviour between filling factors  $\nu = 1$  and  $2$  has been known for several years [1, 2, 3, 4, 5]. Although not always present it has been observed by several groups in samples from more than one source. Similar behaviour has also been seen recently [6] near filling factors  $5/2$  and  $4$ .

There is, as yet, no generally accepted explanation for this phenomenon. Two possibilities are that the insulating phase (IP) might be attributed to a premature and re-entrant Hall insulating state [5] or that it results from a combination of the unusual ordering of the Landau levels and a long-range modulation of the potential [2]. An alternative suggestion [4] is that the IP appears when the large, exchange enhanced, Zeeman splitting causes adjacent Landau levels to cross and could be associated with an unusual spin texture or domain structure. Results are presented here to support this view. It is shown (a) that the behaviour of the Hall resistivity in the IP is not consistent with a Hall insulating state and (b) that although no exotic spin texture states are expected [7] at  $\nu = 2$  this is not necessarily the case for non-integer filling factors, particularly when disorder and screening are taken into account.

## II. EXPERIMENT

Figure 1 shows details of the temperature dependence of the Shubnikov-de Haas (SdH) oscillations in a p-SiGe sample [8] with a density of  $1.7 \times 10^{15} \text{m}^{-2}$  and mobility of  $1.65 \text{m}^2(\text{Vs})^{-1}$ . The IP which appears at  $\nu = 3/2$  is characterised by fixed points with  $\rho_{xx} \sim h/e^2$  (see also figure 2a). They indicate a metal-insulator transition with scaling behaviour [9] similar to that observed for transitions into the high field ( $\nu > 1$ ) Hall insulating state. Separate, well defined, fixed points associated with Integer Quan-

tum Hall Effect (IQHE) transitions can also be observed. These are shifted somewhat from the expected magnetic fields but the critical values (in units of  $e^2/h$ ) of  $\sigma_{xx}^c = 0.65$  and  $\sigma_{xy}^c = 2.7$  for the  $3 \rightarrow 2$  transition and  $\sigma_{xx}^c = 0.59$ ,  $\sigma_{xy}^c = 1.6$  for the  $2 \rightarrow 1$  transition, are close to the expected values of (0.5, 2.5) and (0.5, 1.5) respectively. Near  $B = 3$  tesla the increase of  $\rho_{xx}$  with temperature is metallic-like, mirroring the behaviour around 4 tesla. This is interpreted as the precursor to another IP which is suppressed by the onset of the  $\nu = 2$  IQHE plateau before becoming fully developed. (In a lower density sample this peak becomes fully insulating [6]).

A feature of the data is the behaviour of  $\rho_{xy}$  through the IP. This is not an easy measurement, because any sample inhomogeneity produces spurious  $\rho_{xx}$  terms which can be an order of magnitude larger than the genuine signal, but when these are removed by averaging data with normal and reversed magnetic fields, it can be seen that the Hall resistance tends to vanish as the insulating behaviour strengthens. This is a general feature of the IP: figure 2b shows measurements in a slightly lower density sample and similar trends can also be identified in other data in the literature [3, 5].

A vanishing Hall resistance is incompatible with the  $\nu = 3/2$  IP being a re-entrant Hall insulating state. The Hall insulator [10] appears when the Fermi energy lies in the last Landau level: in the IQHE regime it is characterised by a Hall conductance that vanishes as  $\sigma_{xx}^2$  so the Hall resistivity remains approximately quantised at  $h/e^2$ , in the fractional QHE regime it increases approximately as  $B/n_s e$  (where  $n_s$  is the density) with FQHE plateaus appearing over limited field ranges. The vanishing of  $\rho_{xy}$  therefore indicates the insulating phase is neither an IQHE nor a FQHE Hall insulator.

Coulomb interaction and exchange effects are large in these p-SiGe samples. For example, for that shown in figure 1, at  $\nu = 2$ , the exchange energy  $[(\pi/2)^{1/2}(e^2/\kappa l_m)]$  where  $l_m$  is the magnetic length] is 10 meV, approximately five times larger than the cyclotron energy ( $\hbar\omega_c$ )

of 1.9 meV. The (bare) spin splitting in p-SiGe is relatively large, of order  $\hbar\omega_c/2$ , and is sufficiently enhanced by exchange that the  $0 \uparrow$  and  $1 \downarrow$  Landau levels cross and the system becomes ferromagnetically polarised at  $\nu=2$  (see figure 3). Activation measurements in other p-SiGe samples [4] confirm this is so. At integer filling factors, when the Fermi energy lies in localised states between Landau levels, screening is unimportant, but for non-integer filling factors, when the density of extended states at the Fermi energy is large, it can significantly reduce the exchange enhancement of the spin splitting. A model calculation is presented that explores this situation.

### III. MODEL CALCULATION

The calculation uses the formalism given by Yarlagadda [11] with the screening of the bare coulomb potential ( $V_q$ ) given by

$$\epsilon(q) = 1 + V_q(D_0^\dagger + D_1^\dagger + \Pi^{interLL}) \quad (1)$$

where  $D_N^\sigma$  is the density of states at the Fermi level of the  $N\sigma$  Landau level and  $\Pi^{interLL}$  is an inter-Landau level term. For any significant density of states at the Fermi energy this last term is relatively small and will be ignored. The separation of the  $1 \downarrow$  and  $0 \uparrow$  Landau levels is then given by

$$\Sigma_1^\downarrow - \Sigma_0^\uparrow = \hbar\omega_c - g\mu_B B + \sum_q E_{ex}(q) \quad (2)$$

where

$$E_{ex}(q) = \frac{V_q}{\epsilon(q)}(J_{00}^2 n_0^\uparrow - J_{11}^2 n_1^\downarrow - J_{10}^2 n_0^\downarrow) \quad (3)$$

with  $J_{NM}(ql_m)$  the coupling between the N and M Landau levels and  $n_0^\uparrow$  etc partial filling factors. One can also calculate the total (Hartree Fock) energy of the system ( $E^{HF}$ ) as the integral over occupied states of  $E_{KE} - \frac{1}{2}E_{ex}$  where  $E_{KE}$  is the kinetic energy.

Realistic parameters for the sample shown in figure 1 were used: an effective mass of  $0.2m_e$ , a bare Zeeman splitting of  $0.65 \hbar\omega_c$  and a (Gaussian) Landau level broadening given, according to the self consistent Born approximation, by  $(e\hbar^2\omega_c/2\pi m^*\mu_q)^{1/2}$  where the (measured) quantum mobility  $\mu_q$  is  $1.7\text{m}^2(\text{Vs})^{-1}$ .

Figure 4 shows the separation between the  $\Sigma_1^\downarrow$  and  $\Sigma_0^\uparrow$  Landau levels as a function of  $n_0^\uparrow$ , determined (a) from eqns. 2 and 3 and (b) from the constraint that  $n_0^\downarrow + n_0^\uparrow + n_1^\downarrow = \nu$ . It is assumed the lowest,  $0^\downarrow$ , level always remains full. Valid solutions occur at the intersection of these two curves. At  $\nu=2$ , the calculation without screening (see figure 4a) shows an unstable, degenerate solution and two stable solutions corresponding to either

a ferromagnetic or paramagnetic ordering of the Landau levels (the former with the lowest energy). This essentially reproduces the result of Giuliani and Quinn [7] who also showed that for non-degenerate Landau levels one of these stable solutions will pre-empt any spin-density-like state.

When screening is included the ferromagnetic configuration becomes even more strongly favoured but the other solutions are significantly altered, in particular the paramagnetic solution now involves degenerate Landau levels and there is an increase in the total energy. When the filling factor moves away from two more significant differences appear (see figure 4b). The density of states at the Fermi energy is always non-zero, screening becomes important and the exchange enhancement of the spin splitting is reduced. Under these conditions only one, degenerate, solution remains and the ferromagnetically polarised state can no longer exist.

Figure 5 shows the Landau level separation as a function of filling factor. The ferromagnetically polarised state appears only at integer filling factors, when the screening is small. Elsewhere, the levels stay close together and are degenerate with a predominantly paramagnetic alignment. The transition into the ferromagnetically polarised state is sharp near  $\nu=2$  but much smoother near  $\nu=1$  and 3.

### IV. DISCUSSION

The model calculation can explain the existence of both a high field metal/insulator transition and separate, well defined, IQHE transitions. Near integer filling factors, the exchange interaction ensures the Landau levels are well separated and the IQHE transitions can then occur in the tail of an isolated Landau level. Away from integer filling factors, screening means the Landau levels overlap which correlates with the appearance of the insulating (or metallic) behaviour. Separate metal/insulator and IQHE transitions occur because the Landau levels realign as the filling factor changes. In some circumstances, for example in lower mobility samples or at higher temperatures, this realignment may not occur and the high field IP and IQHE transitions will then merge and no longer be distinguishable.

Away from integer filling factors the only allowed solution corresponds to degenerate Landau levels (at least for a homogeneous system). Although the Hartree-Fock energy in figure 4b appears relatively flat as a function of the relative populations of the two levels, a detailed examination shows there could be a small lowering of the energy if the system were to break up into domains. While the exact nature of such a state (or some other kind of spin texture) is uncertain such a mechanism would appear to be a candidate for the cause of the insulating behaviour.

In high mobility, GaAs based, 2DEG samples, coulomb interaction effects manifest themselves as fractional

quantum Hall effect features. Because of the difference in effective masses ( $0.2m_e$  compared with  $0.067m_e$ ) and the ratio of the transport to quantum lifetimes (approximately one in p-SiGe compared with 20 or so in n-GaAs 2DEGs) the Landau level broadening in the p-SiGe sample investigated here is very similar to that of a GaAs based 2DEG with a mobility of over  $200 \text{ m}^2(\text{Vs})^{-1}$ . The exchange energy depends only on magnetic field so the absence of fractional quantum Hall features is a little puzzling. The essential difference is that in n-GaAs heterostructures higher Landau levels are well removed in energy, typically by several times the interaction energy, whereas in p-SiGe there are two degenerate Landau levels at the Fermi energy. This is consistent with the insulating phase (and associated high field metal-insulator transition) being a direct consequence of strong exchange interactions in degenerate Landau levels suppressing more usual FQHE behaviour.

The model calculation does not include a number of factors such as: the finite thickness of the hole gas, correlation effects, the existence of a mobility edge and the contribution of inter-Landau level screening but the result, that screening causes the Landau levels to stick together, is robust. A negative feed-back mechanism (relating the exchange splitting of the Landau levels to the screening) ensures that the separation of the levels is small and relatively independent of exact values of the parameters, provided only that the *unscreened* exchange energy is large compared with the bare separation and that screening significantly reduces this exchange energy.

In electron systems Landau level crossings occur and can be tuned with parallel magnetic fields [12]. They are identified by ‘spikes’ in the Shubnikov-de Haas data with, in some cases, hysteresis. This is interpreted in terms of domain formation in these quantum Hall Ising ferromagnets. The question of spin textures in such systems has recently been studied theoretically by Brey and Tejedor

[13]. They find that a spin-orbit interaction, which directly couples the  $0 \uparrow$  and  $1 \downarrow$  states, provides a means of producing a gap in the energy spectrum of the charged excitations that cross domain walls. It is therefore interesting to note that in p-SiGe, where strain separates the two degenerate heavy-hole states at the Brillouin zone centre, there is a large spin-orbit coupling with the holes having almost pure  $|M_J| = 3/2$  character. However, it should be also noted these systems are not necessarily directly equivalent to that discussed here which often involve relatively weakly coupled 2DEGs, either in a rather wide single quantum well or in two wells separated by a barrier.

## V. CONCLUSION

Fixed points defining both IQHE transitions and a high field insulating phase can be separately identified in Shubnikov-de Haas measurements in a p-SiGe sample. Further, it is found that the Hall resistivity in the IP tends towards zero as the temperature is lowered, which implies the IP is not a re-entrant Hall insulator.

A model calculation is presented which explains these two types of behaviour in terms of the large exchange interaction which is screened when the Fermi energy lies within a Landau level but not when it lies in localised states between Landau levels. Insulating behaviour (or more generally a metal-insulator transition) co-incides with the overlap of degenerate Landau levels overlap at the Fermi energy. It is suggested therefore that the insulating behaviour could well be a direct consequence some form of domain structure or other kind of spin texture induced by a strong exchange interaction in degenerate Landau levels.

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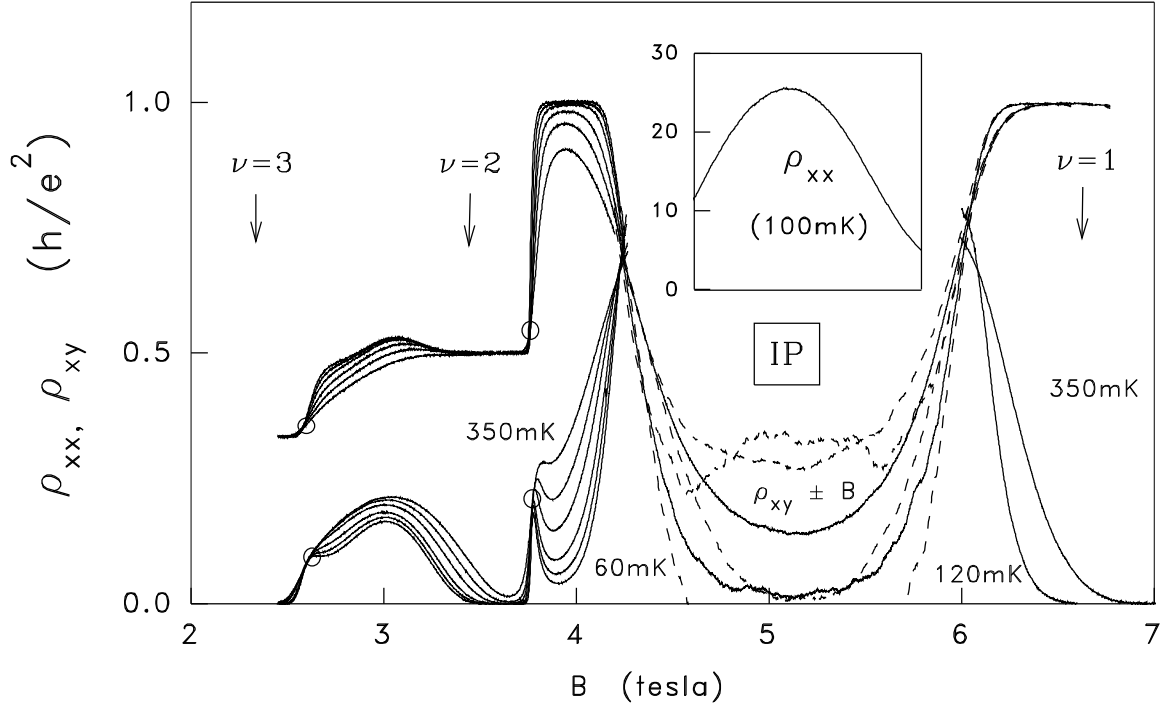


FIG. 1: A composite figure showing Shubnikov-de Haas data for the sample described in the text. The IP appears around 5 tesla. Below 4.3 tesla data is shown for six temperatures between 60 and 350mK: note the fixed points indicating IQHE transitions. Above 4.3 tesla  $\rho_{xy}$  data is shown at temperatures of 120 and 350mK for normal and reversed magnetic fields (dashed) and for the average of these (solid). The inset shows  $\rho_{xx}$ , at 100mK, determined from a 2-terminal measurement.

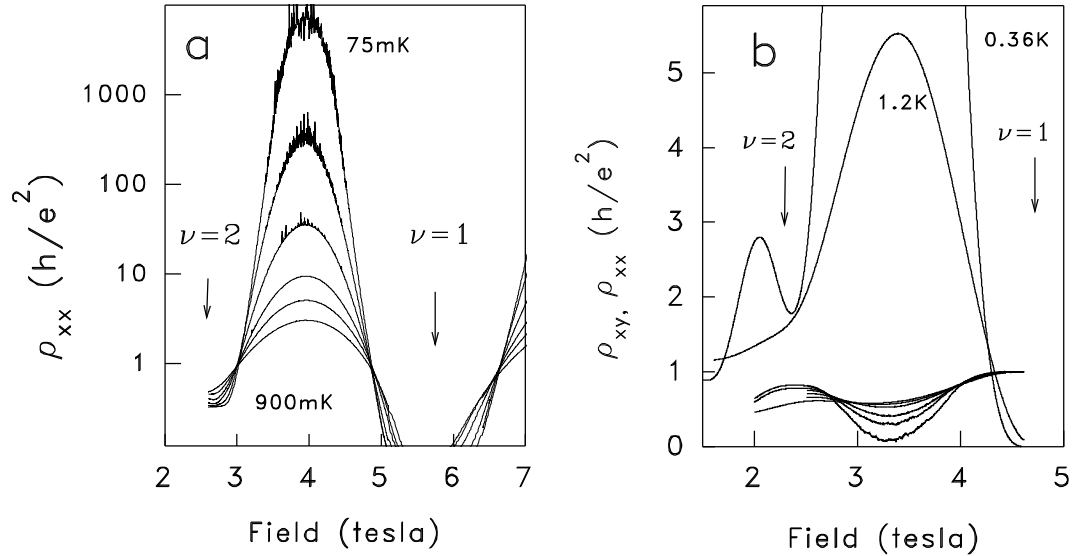


FIG. 2: Shubnikov-de Haas oscillations from two p-SiGe samples with densities of approximately  $1.2 \times 10^{15} \text{ m}^{-2}$ . (a)  $\rho_{xx}$ , determined using a 2-terminal measurement, for temperatures of 75, 120, 220, 400, 600 and 900mK. (b) Values of  $\rho_{xx}$  at 0.36 and 1.20K and  $\rho_{xy}$  (averages for normal and reversed magnetic fields) at 0.27, 0.36, 0.45, 0.65, 0.90 and 1.20K.

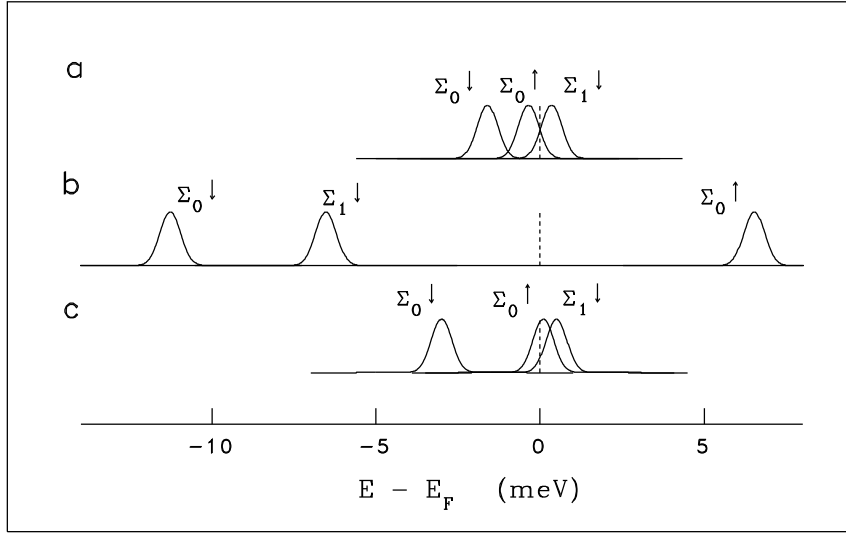


FIG. 3: Alignment of the three lowest Landau levels for sample parameters appropriate to the data shown in figure 1. (a) At  $\nu=2$ , using the bare value,  $g\mu_B B$ , for the spin splitting. (b) At  $\nu=2$  with exchange enhanced spin splitting according to eqn. 2. (c) At  $\nu=3/2$  with a screened exchange term.

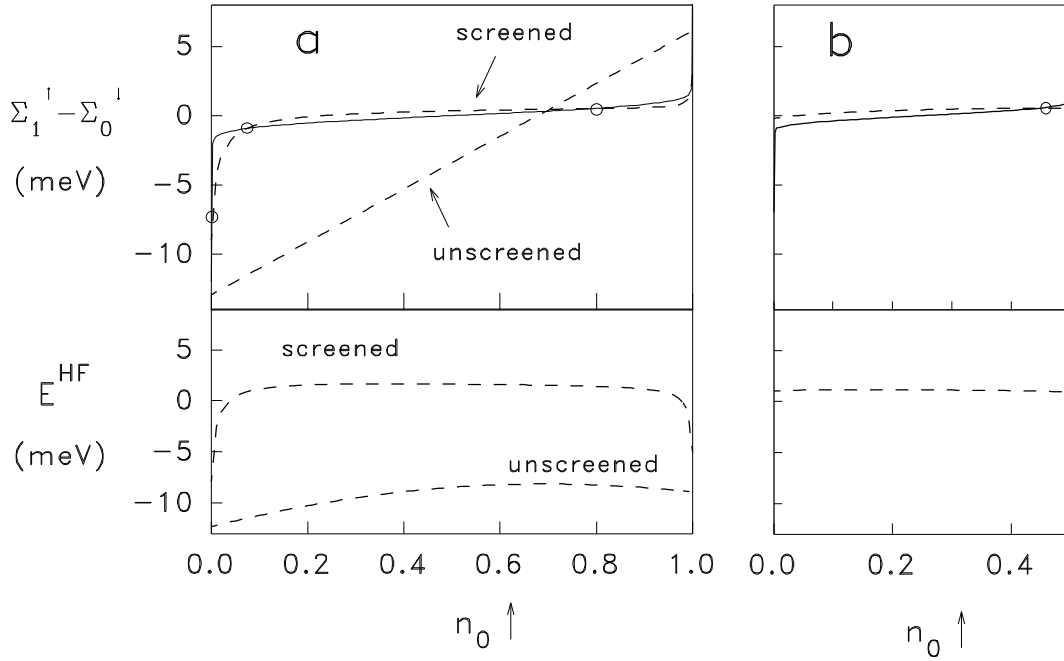


FIG. 4: Landau level spacing and total (Hartree-Fock) energy as a function of partial filling factor  $n_0^\uparrow$ . Dashed lines correspond to eqns. 2 and 3, solid lines to the requirement that the sum of the partial filling factors must equal  $\nu$ . (a) at filling factor  $\nu=2$ , with and without screening. Circles show the three possible solutions when screening is included. (b) at filling factor 1.5, in this case only one solution is allowed.

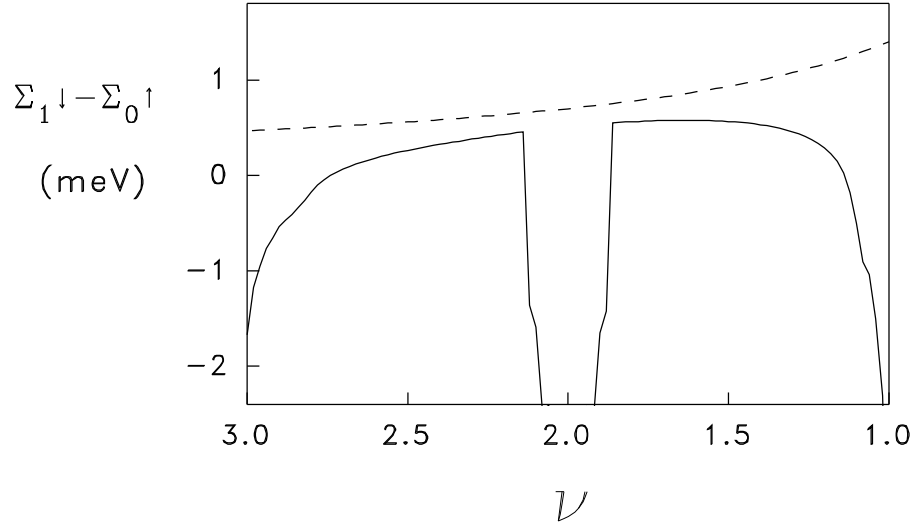


FIG. 5: Calculated Landau level separation as a function of filling factor. Dashed curve shows the bare value ie  $\hbar\omega_c - g\mu_B B$ . For comparison the width of a Landau level (full-width at half maximum) is 0.74meV at  $\nu=2$  and varies as  $B^{1/2}$ .